Forward and Inverse Kinematic equations:

## Orientation (sec. 2.10 Book)

This can be accomplished by rotating about the current frame axes.
The appropriate sequence of rotations depend on the design of the wrist of the robot and the way the joints are assembled together.
We will consider the following three common configurations:

(a) Roll, Pitch, Yaw (RPY) angles.

(b) Euler angles.

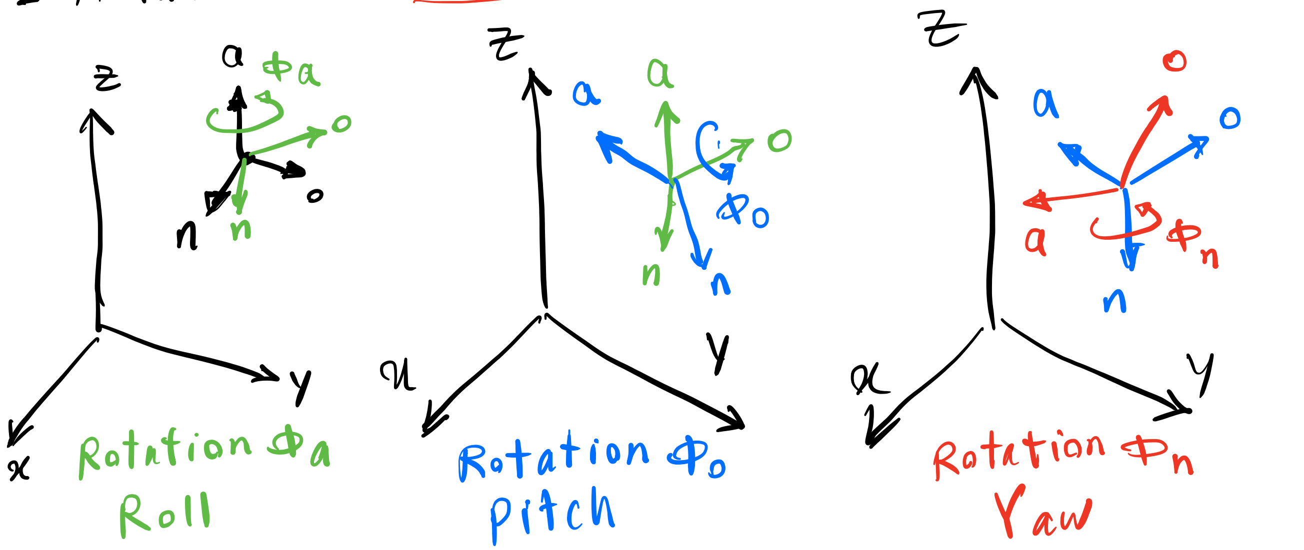
(c) Articulated joints.

# Roll, pitch, Yaw (RPY) Angles

This is a sequence of three rotations about current a-, o-, and n-axes, respectively, which will orient the hand of the robot to a desired orientation.

the RPY sequence of rotations consists of:

- Rotation of $\Phi_a$ about the a-axis called Roll,
- Rotation of $\Phi_o$ about the o-axis called Pitch,
- Rotation of $\Phi_n$ about the n-axis called Yaw



The matrix representing the RPY orientation change will be:

$$RPY(\Phi_a, \Phi_o, \Phi_n) = Rot(a, \Phi_a)\, Rot(o, \Phi_o)\, Rot(n, \Phi_n)$$

$$= \begin{bmatrix} C\Phi_a C\Phi_o & C\Phi_a S\Phi_o S\Phi_n - S\Phi_a C\Phi_n & C\Phi_a S\Phi_o C\Phi_n + S\Phi_a S\Phi_n & 0 \\ S\Phi_a C\Phi_o & S\Phi_a S\Phi_o S\Phi_n + C\Phi_a C\Phi_n & S\Phi_a S\Phi_o C\Phi_n - C\Phi_a S\Phi_n & 0 \\ -S\Phi_o & C\Phi_o S\Phi_n & C\Phi_o C\Phi_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is only orientation change. The location and the final orientation of the frame relative to the reference frame will be th product of the two matrices representing the position change and the RPY. For example, suppose that a robot is designed based on spherical coordinates and RPY.

Then the robot may be represented by:

$$ {}^{R}T_{H} = T_{sph}(r, \beta, \gamma) \times RPY(\phi_a, \phi_o, \phi_n) $$

The inverse kinematic solution for the RPY is more complicated than the spherical coordinates because here there are three coupled angles. To solve for sines and cosines, we will have to de-couple these angles. To do this, we will pre-multiply both sides by the inverse of $Rot(a, \phi_a)$ as below.

$$ RPY(\phi_a, \phi_o, \phi_n) = Rot(a, \phi_a)\, Rot(o, \phi_o)\, Rot(n, \phi_n) $$

$$ Rot(a, \phi_a)^{-1}\, RPY(\phi_a, \phi_o, \phi_n) $$

$$ = \underbrace{Rot(a, \phi_a)^{-1}\, Rot(a, \phi_a)}_{I}\, Rot(o, \phi_o)\, Rot(n, \phi_n) $$

$$\mathrm{Rot}(a, \Phi_a)^{-1}\, \mathrm{RPY}(\Phi_a, \Phi_o, \Phi_n) = \mathrm{Rot}(o, \Phi_o)\, \mathrm{Rot}(n, \Phi_n)$$

Assume that the final desired orientation achieved by RPY is represented by the matrix below:

$$\mathrm{RPY}(\Phi_a, \Phi_o, \Phi_n) = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now substitute in the above equation.

$$\mathrm{Rot}(a, \Phi_a)^{-1} \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathrm{Rot}(o, \Phi_o)\, \mathrm{Rot}(n, \Phi_n)$$

Multiplying the matrices, we will get:

$$\begin{bmatrix} n_x C\Phi_a + n_y S\Phi_a & o_x C\Phi_a + o_y S\Phi_a & a_x C\Phi_a + a_y S\Phi_a & 0 \\ n_y C\Phi_a - n_x S\Phi_a & o_y C\Phi_a - o_x S\Phi_a & a_y C\Phi_a - a_x S\Phi_a & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\Phi_o & S\Phi_o S\Phi_n & S\Phi_o C\Phi_n & 0 \\ 0 & C\Phi_n & -S\Phi_n & 0 \\ -S\Phi_o & C\Phi_o S\Phi_n & C\Phi_o C\Phi_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equating the different elements of the right-hand and left-hand sides of the equation above will result in the following:

From the 2,1 elements, we get:

$$n_y C\Phi_a - n_x S\Phi_a = 0$$

$$\Rightarrow n_y c\Phi_a = n_x s\Phi_a$$

$$\frac{s\Phi_a}{c\Phi_a} = \frac{n_y}{n_x}$$

$$\tan\Phi_a = \frac{n_y}{n_x} \qquad \cancel{\Phi_a = \mathrm{Atan}\left(\frac{n_y}{n_x}\right)}$$

$$\Phi_a = \mathrm{ATAN2}(n_y, n_x)$$

Calculate $\mathrm{Atan}\left(\frac{n_y}{n_x}\right)$ but also tak into acount the signs for $n_y$ and $n_x$ when calculating the angle.

$$\Phi_a = \mathrm{ATAN2}(-n_y, -n_x)$$

study sectian 2.16