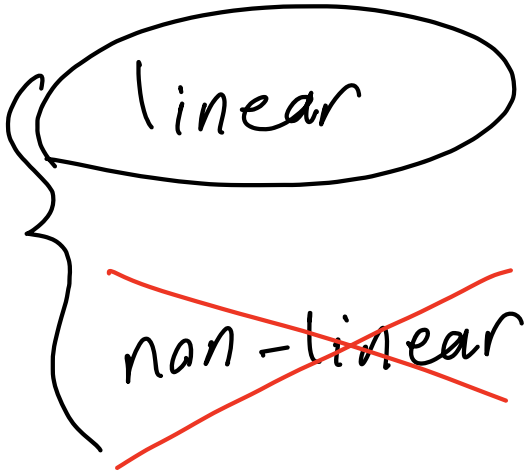
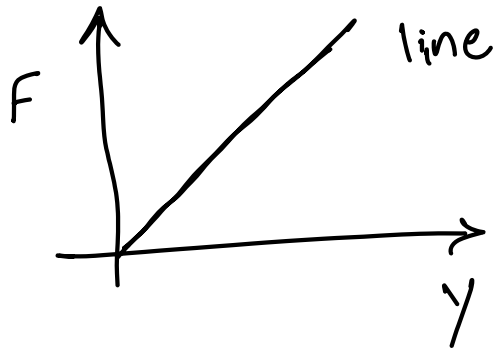
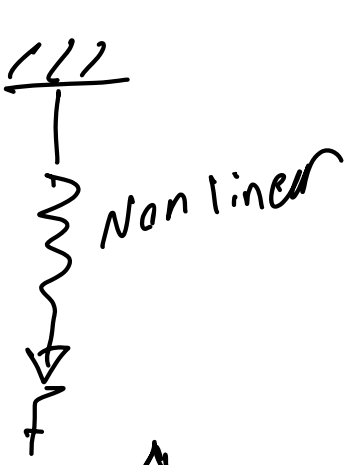
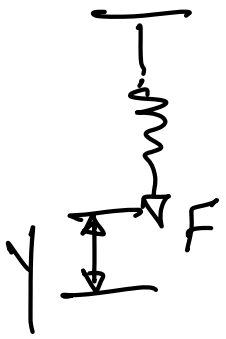
vibrations → Oscillatory motion

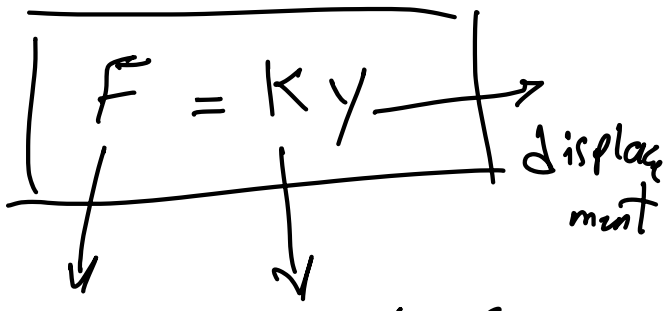




spring



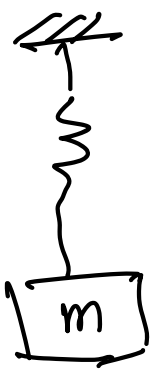




Force

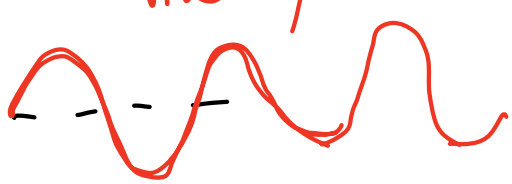
Stiffness of the spring

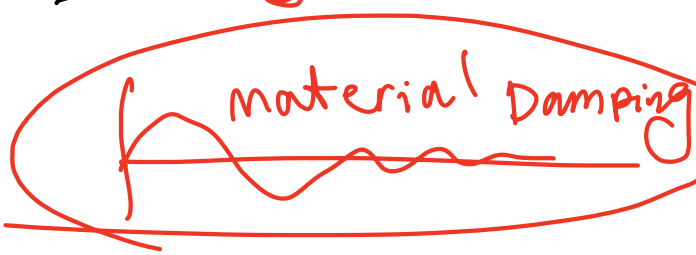
Forced vibration



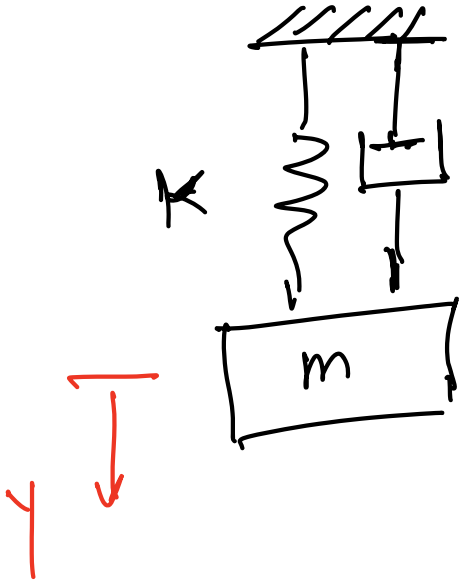
Free vibration

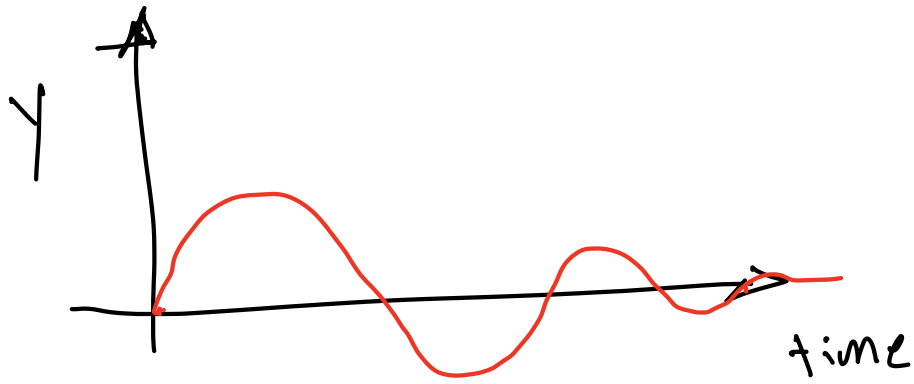
Theory



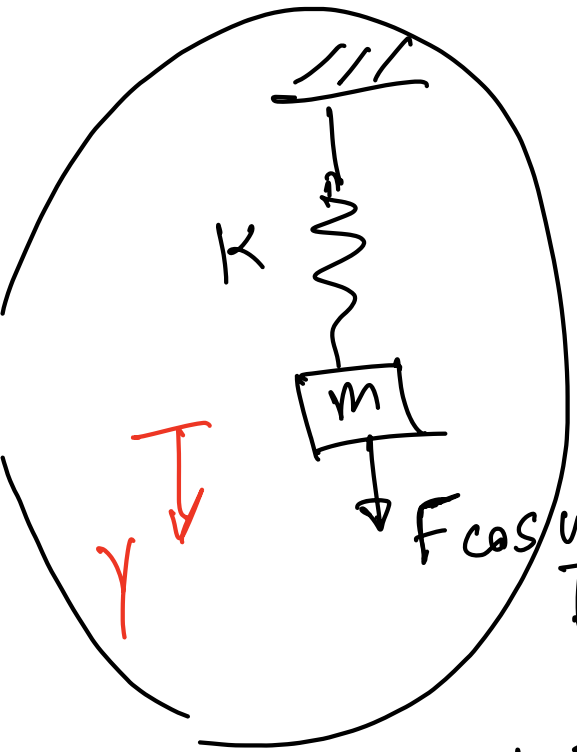


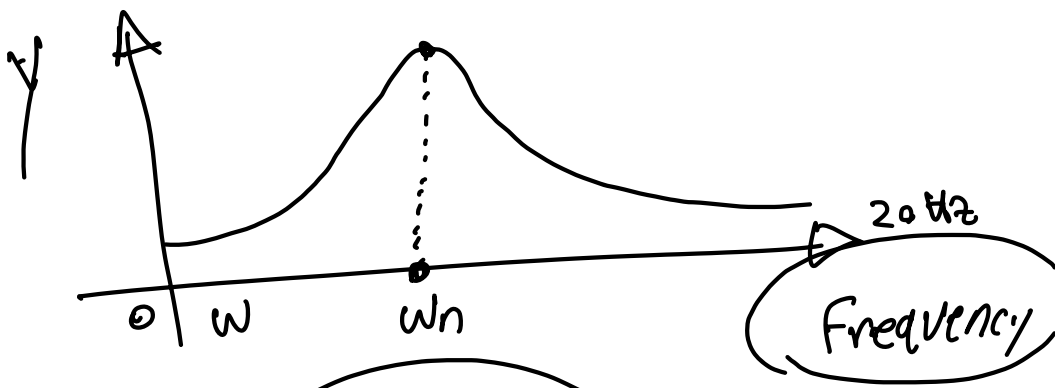
Damped vibration





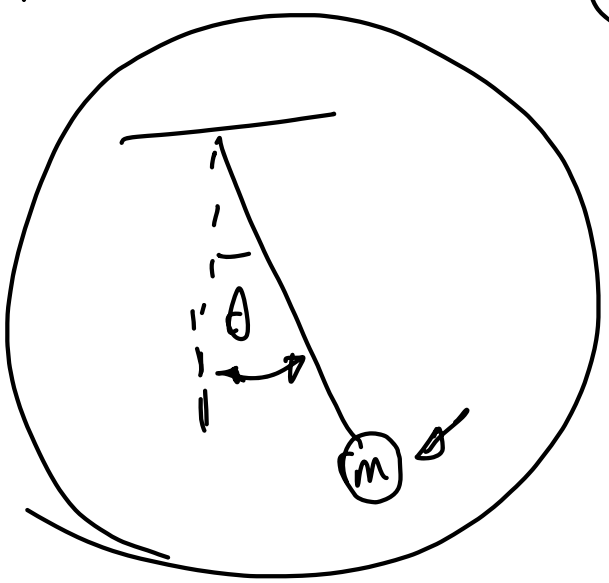
Natural Frequency

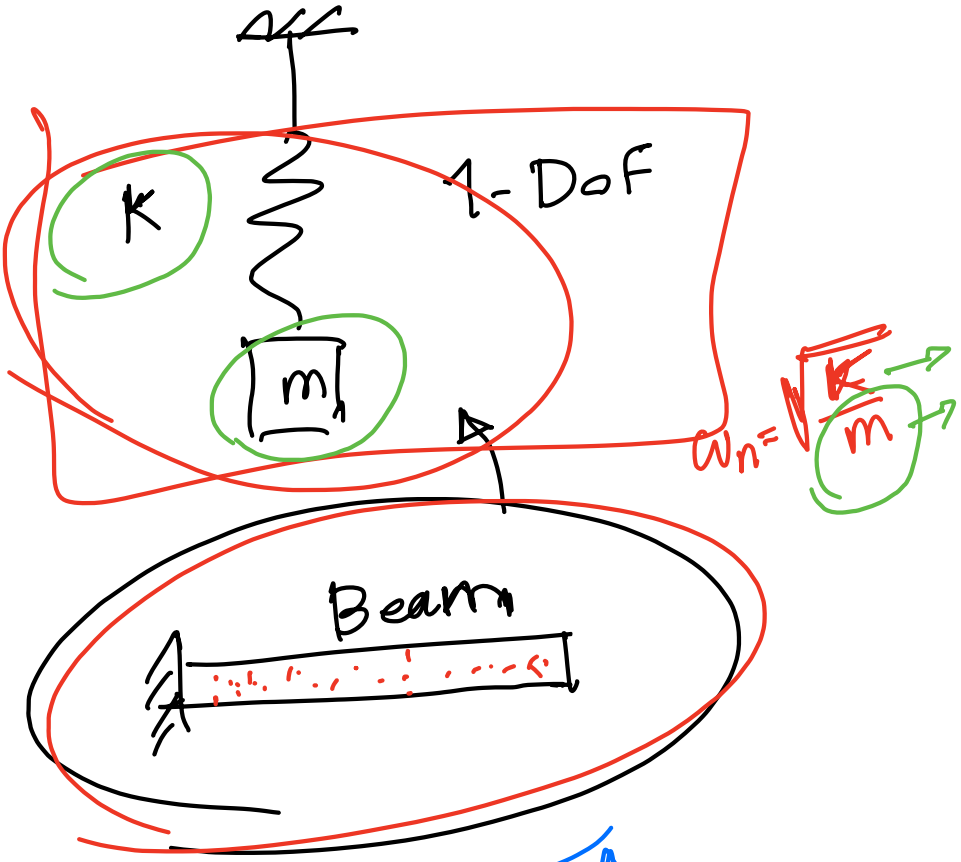


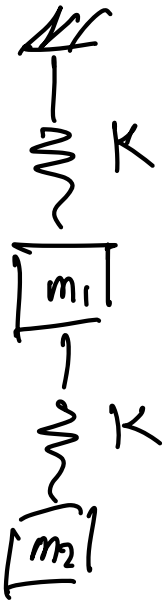


$$w = w_n = \sqrt{\frac{K}{m}}$$

natural frequency

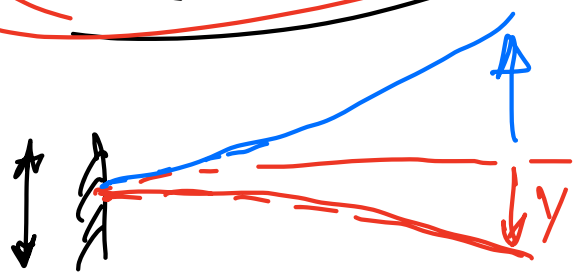






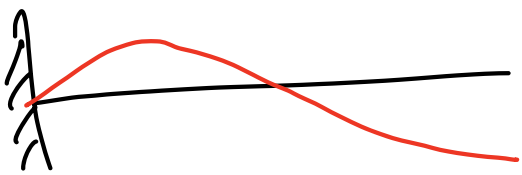
2DoF

mode shape

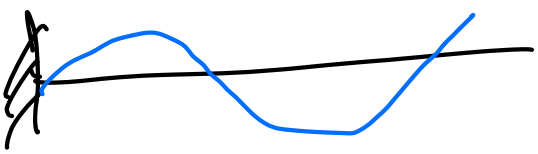


1st natural frequency

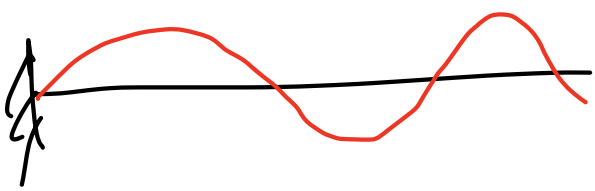
lower frequency



2nd natural frequency

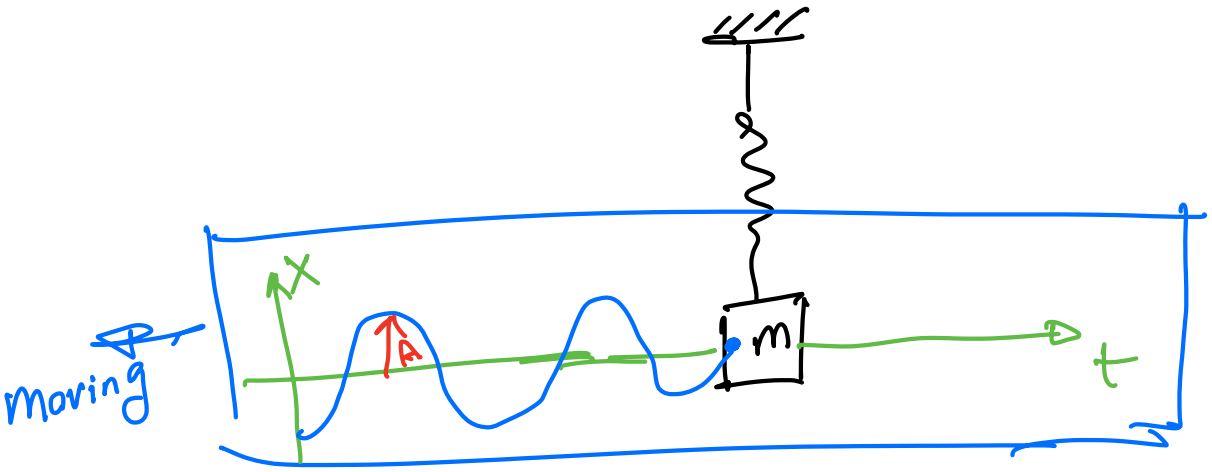


3rd



4th

Harmonic Motion



$$X = A \sin \omega t$$

f = frequency (Hz) τ = period of oscolation (Second)

$$f = \frac{1}{\tau}$$

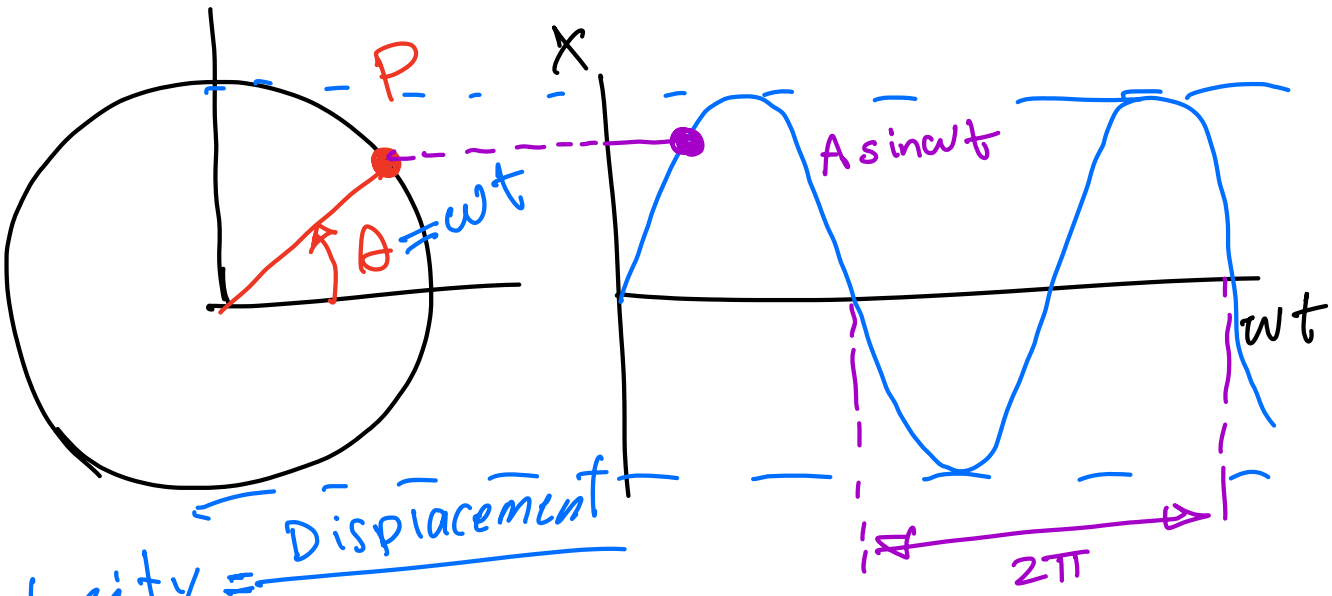
ω = angular frequency

$$\omega = 2\pi f$$

unit → rad/s

$$\omega = \frac{2\pi}{\tau}$$

$$X = A \sin 2\pi \frac{t}{\tau}$$



$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$V = \frac{d}{t}$$

$$\text{angular velocity} = \frac{\text{angular Displacement}}{\text{Time}}$$

$$\omega = \frac{\theta}{t}$$

Displacement $\quad X = A \sin \omega t \quad (1)$

Velocity $\quad \dot{X} = \frac{dx}{dt} = \omega A \cos \omega t \quad (2)$

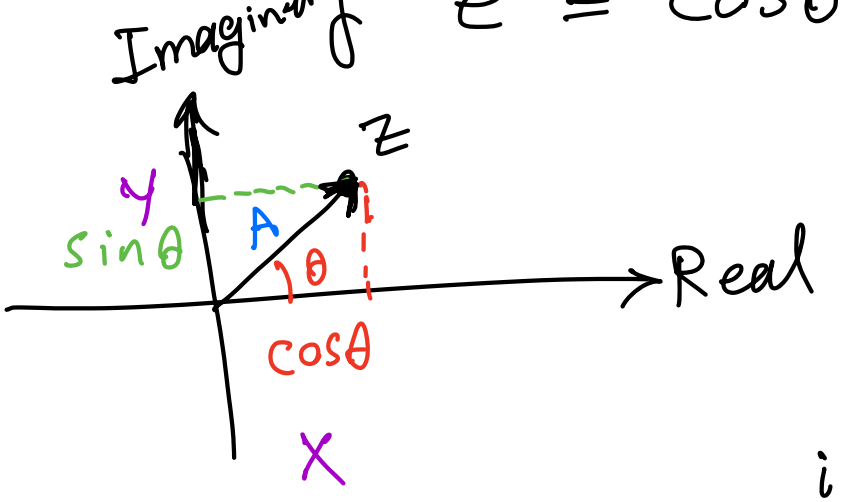
Acceleration $\quad \ddot{X} = \frac{d^2x}{dt^2} = -\omega^2 A \sin \omega t \quad (3)$

(1) & (3) $\Rightarrow \quad \ddot{X} = -\omega^2 X$

Exponential form:

Euler's equation

$$e^{i\theta} = \cos\theta + i\sin\theta$$



$\theta = \omega t$

$$Z = Ae^{i\omega t}$$

$$= A\cos\omega t + iA\sin\omega t$$

$A\cos\omega t$: x ; $iA\sin\omega t$: y

$$= x + iy$$

---

conjugate of z :

$$z^* = Ae^{-i\omega t} = A(\cos\omega t - i\sin\omega t)$$

$$X = \frac{1}{2}(z + z^*) = A\cos\omega t$$

Real part

Rules:

$$z_1 = A_1 e^{i\theta_1} \qquad z_2 = A_2 e^{i\theta_2}$$

$$z_1 z_2 = A_1 A_2 e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \left(\frac{A_1}{A_2}\right) e^{i(\theta_1 - \theta_2)}$$

$$z^n = A^n e^{in\theta}$$