Ziegler–Nichols PID tuning method

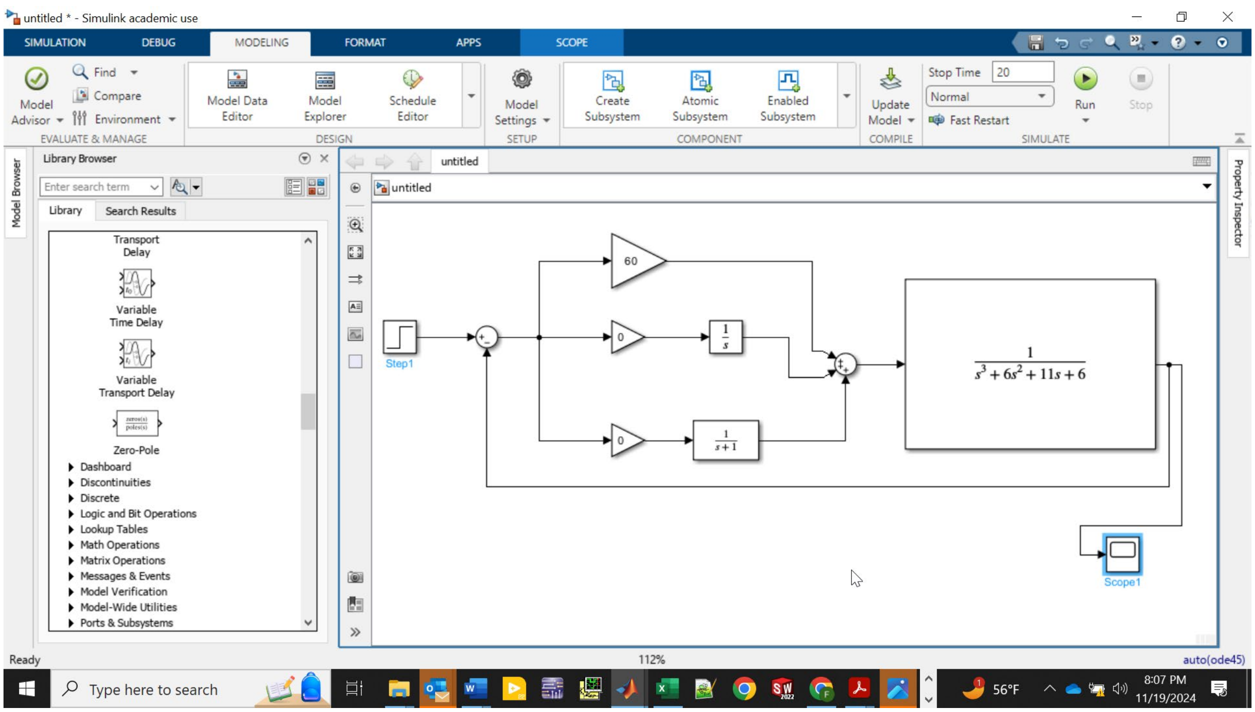
Use the Ziegler–Nichols tuning method to obtain the control gains for the system below:

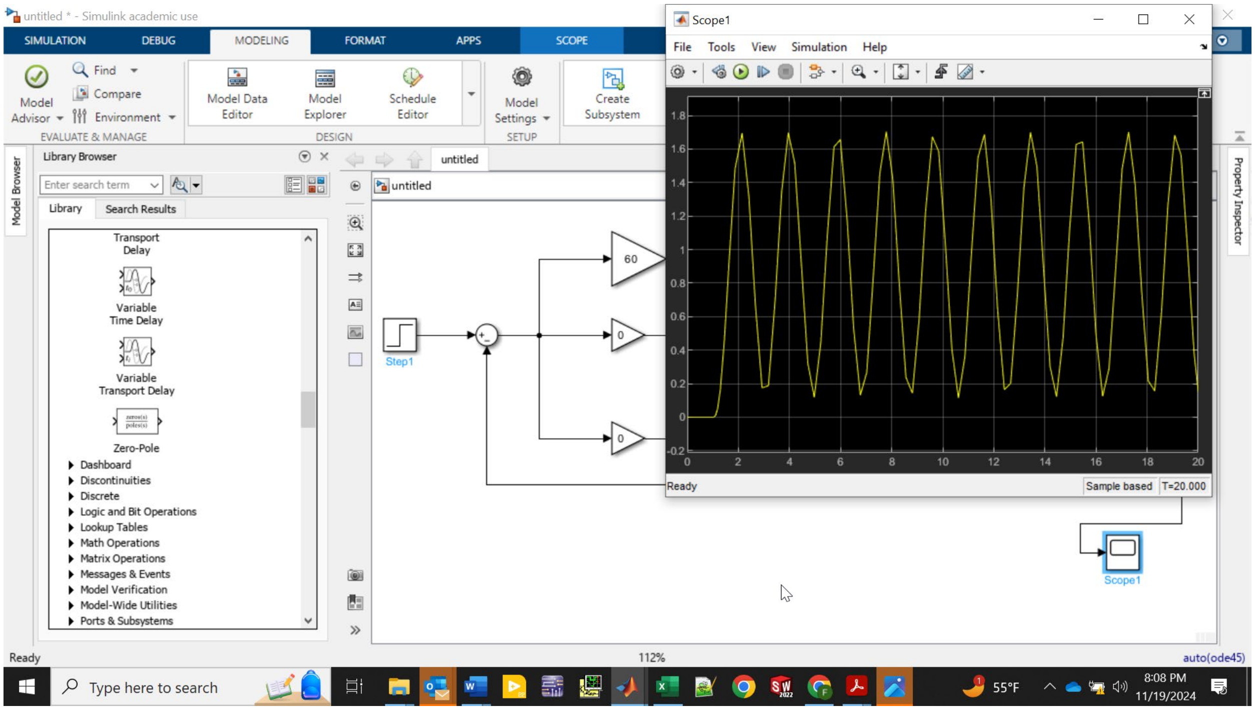
$$\frac{1}{s^3 + 6s^2 + 11s + 6}$$

-Start with the Kp only (Ki and Kd are zero)

-Start with a small value for Kp (for example Kp=1).

-Increase Kp until the system is on the verge of stability (oscillates with a constant amplitude without settling down)





This proportional control gain is called the ultimate gain Ku:

Ku=60

Find the period of the oscillation corresponding to Ku.

Tu=2 seconds

Ziegler–Nichols method[1]

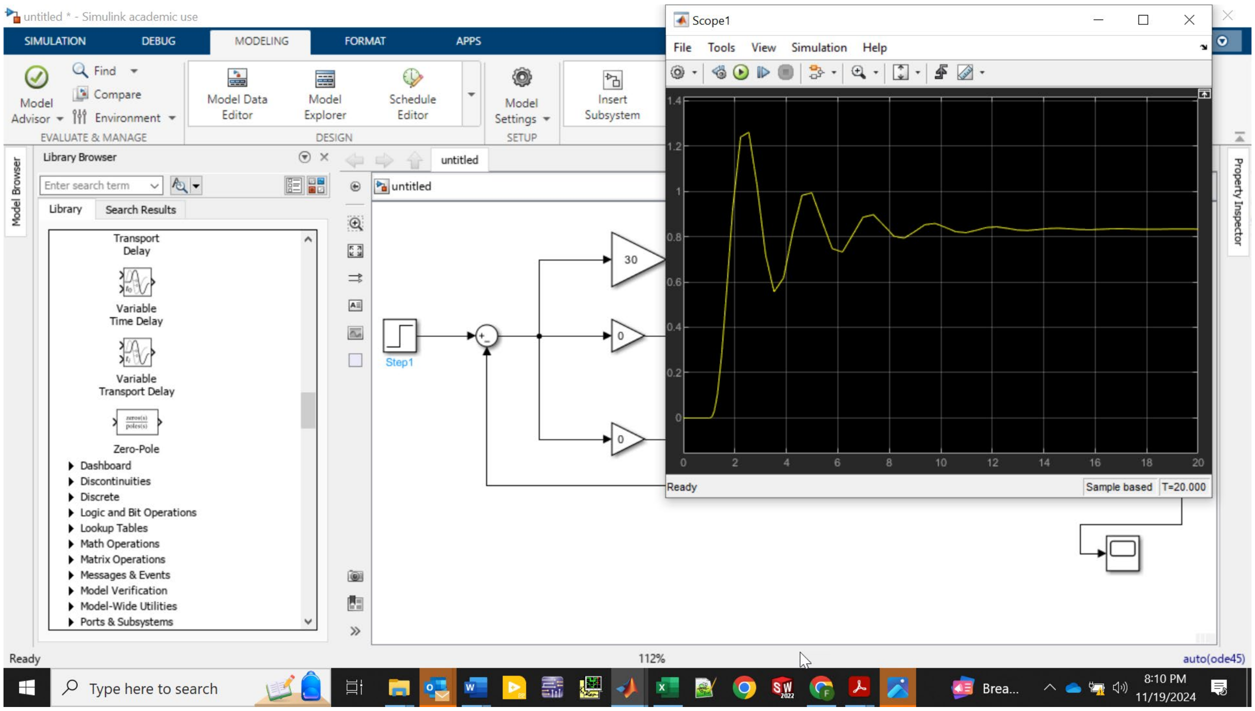
Control Type	K_p	T_i	T_d	K_i	K_d
P	$0.5K_u$	–	–	–	–
PI	$0.45K_u$	$0.8\overline{3}T_u$	–	$0.54K_u/T_u$	–
PD	$0.8K_u$	–	$0.125T_u$	–	$0.10K_uT_u$
classic PID[2]	$0.6K_u$	$0.5T_u$	$0.125T_u$	$1.2K_u/T_u$	$0.075K_uT_u$
Pessen Integral Rule[2]	$0.7K_u$	$0.4T_u$	$0.15T_u$	$1.75K_u/T_u$	$0.105K_uT_u$
some overshoot[2]	$0.3\overline{3}K_u$	$0.50T_u$	$0.3\overline{3}T_u$	$0.6\overline{6}K_u/T_u$	$0.1\overline{1}K_uT_u$
no overshoot[2]	$0.20K_u$	$0.50T_u$	$0.3\overline{3}T_u$	$0.40K_u/T_u$	$0.06\overline{6}K_uT_u$

https://en.wikipedia.org/wiki/Ziegler%E2%80%93Nichols_method

Control type: P

0.5 Ku=30

Update the Kp gain and run the simulation:



Steady-state error exists.

Now use the Integral controller:

Ziegler–Nichols method[1]

Control Type	K_p	T_i	T_d	K_i	K_d
P	$0.5K_u$	–	–	–	–
PI	$0.45K_u$	$0.8\overline{3}T_u$	–	$0.54K_u/T_u$	–

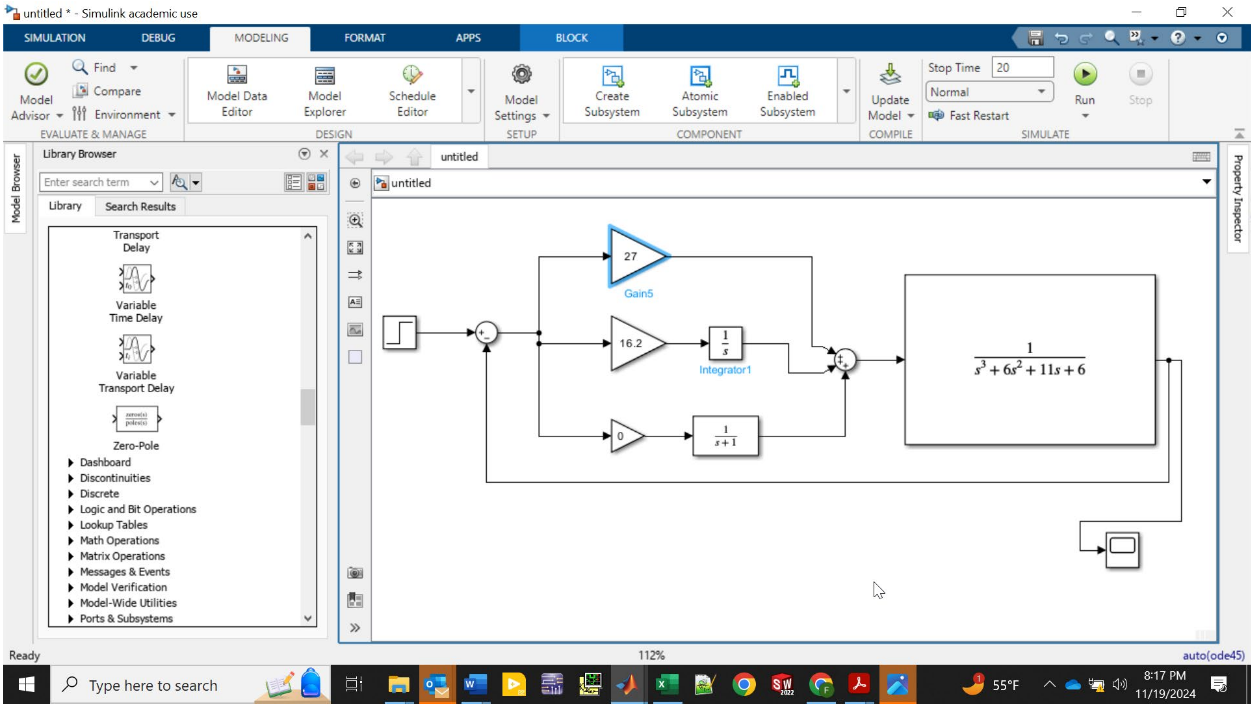
https://en.wikipedia.org/wiki/Ziegler%E2%80%93Nichols_method

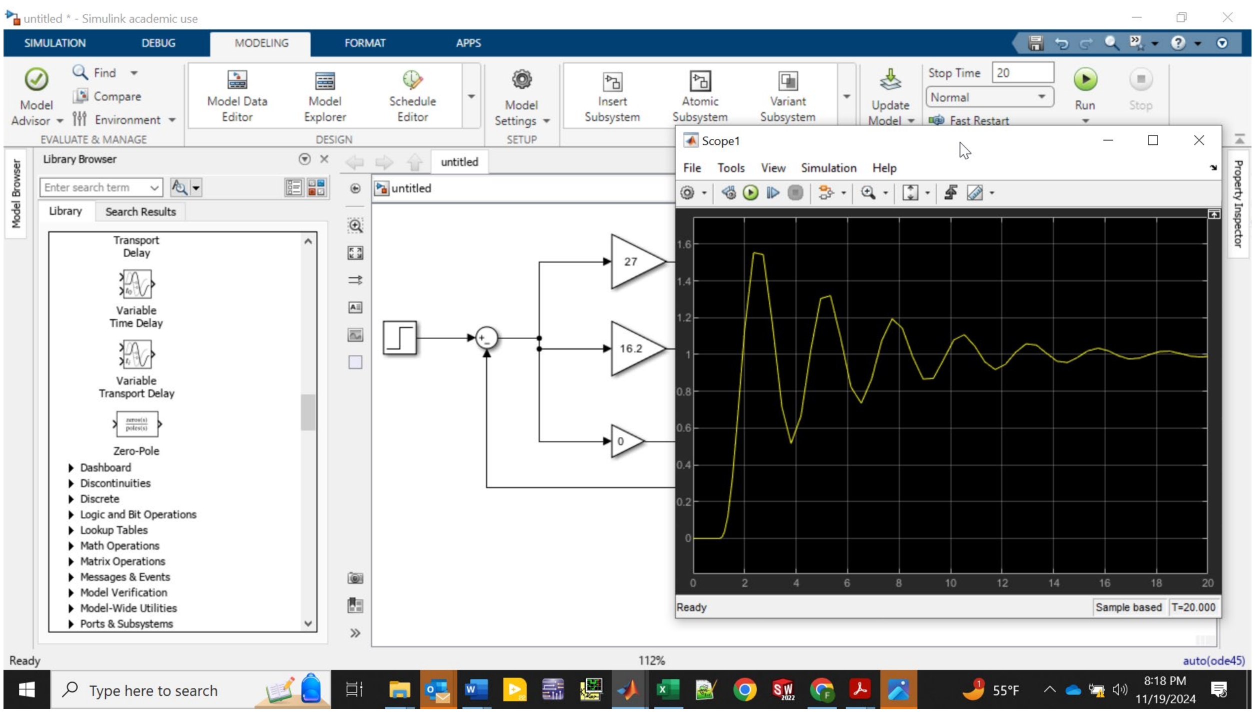
Control type: PI

Kp = 0.45 Ku = 27

Ki = 0.54Ku/Tu = 16.2

Update the Kp and Ki and run the simulation:





The overshoot can be reduces using the derivative controller.

PID controller:

Note: The derivative controller can not be used on it's own. It requires a filter to remove the high frequency noise (also a derivative term is predicting the future which is not realistic in a practical situation). Therefore a filter has to be added to the derivative controller. The derivative controller with the filter looks like: N.S/(S+N)

In this example we are using a filter with N=100 (to remove the frequency higher than 100 rad/s). Therefore and the derivative controller with the filter looks like: 100S/(S+100).

Ziegler–Nichols method[1]

Control Type	K_p	T_i	T_d	K_i	K_d
P	$0.5K_u$	–	–	–	–
PI	$0.45K_u$	$0.8\bar{3}T_u$	–	$0.54K_u/T_u$	–
PD	$0.8K_u$	–	$0.125T_u$	–	$0.10K_uT_u$
classic PID[2]	$0.6K_u$	$0.5T_u$	$0.125T_u$	$1.2K_u/T_u$	$0.075K_uT_u$

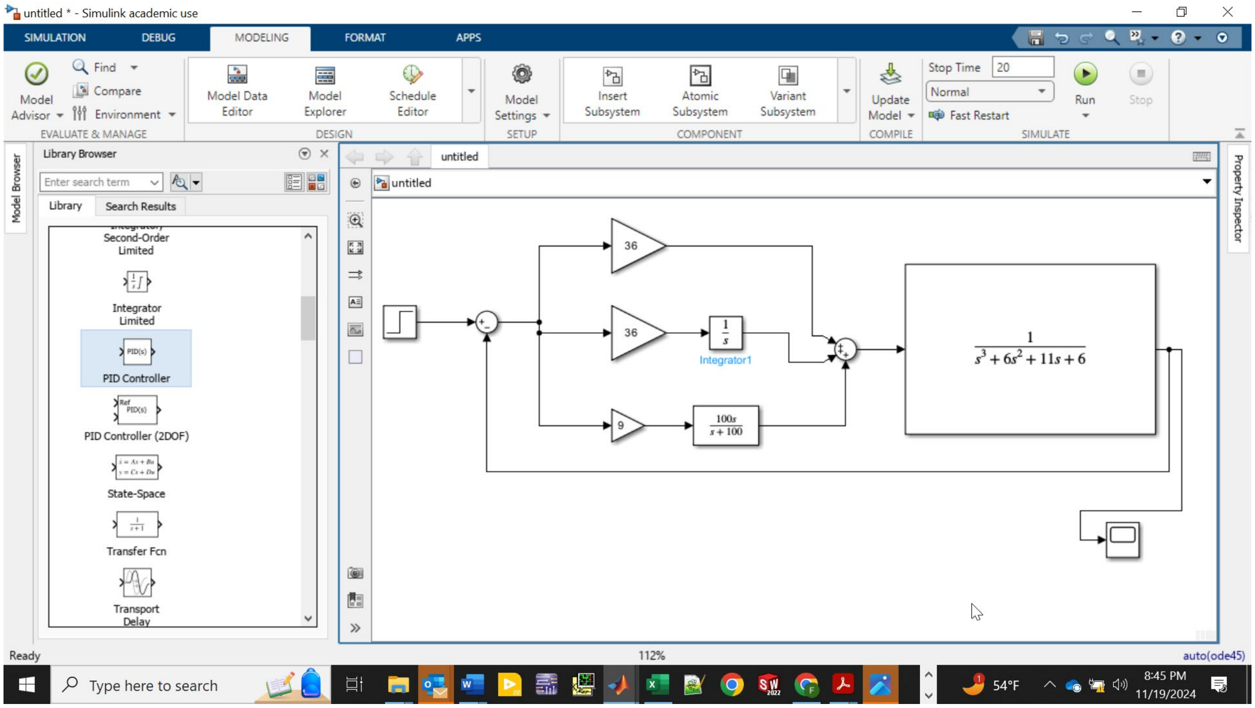
https://en.wikipedia.org/wiki/Ziegler%E2%80%93Nichols_method

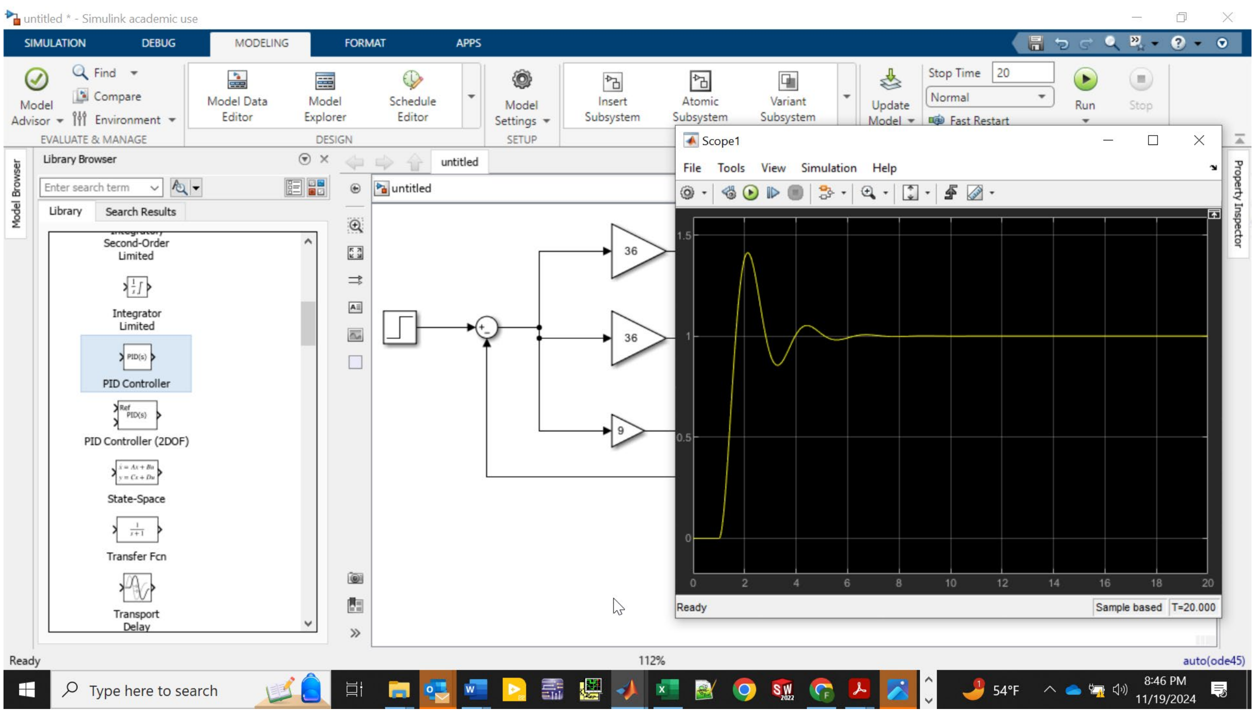
Control type: PID

Kp = 0.6 Ku = 36

Ki = 1.2Ku/Tu = 36

Kd = 0.075 Ku Tu = 9





These initial values of the controller gains for Kp, Ki, and Kd are the starting values for the tuning. Further tuning can be performed by trial and error (after this initial tuning) to improve the response and fine tuning the response, if needed.